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Importance Analysis of the Aircraft Flap Mechanism Movement Failure

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Importance measure is used to describe effect of basic event on top event in fault tree, and the basic events can be ranked using the importance measures in probability risk assessment. By taking uncertainty of the basic event probability into account, a new importance measure is proposed on the traditional one. For complicated engineering systems such as aircraft flap mechanism, the probability of the basic event is usually uncertain due to some uncontrollable factors, and this uncertainty may have significant effect on the importance measure of the basic event. The traditional importance measure employs the assumption of the certain probabilities of the basic events, hence the basic events cannot be properly ranked in the case of the uncertain probability. Contrarily, the new importance measure is on the assumption of the uncertain probability of the basic event. By considering the distribution of the traditional importance measure corresponding to the actual uncertainty of the basic event probability, and using a characteristic value, usually the expectation of the importance measure distribution, to describe the effects, the new importance measure can properly rank the basic events with the uncertainty probability. After the probability density evolution method is employed to solve the new importance measure, the basic event importance is analyzed for the unilateral asymmetric movement failure of the aircraft flap mechanism. The results show that the new importance measure solution based on probability density evolution method is efficient under the acceptable precision. Neglect of the actual uncertainty of the basic event probability may introduce error rank, which is demonstrated by the comparison of the illustrations.

Nomenclature

DIM_i	=	the differential importance measure of X_i
$\mathrm{DIM}_i^{H_1}$	=	the differential importance measure of X_i in case H_1
$\mathrm{DIM}_i^{H_2}$	=	the differential importance measure of X_i in case H_2
$f_{R_i}(R_i)$	=	the probability density function of the importance order R_i of X_i
I_B^i	=	the Birnbaum importance measure of X_i
I_{cr}^i	=	the criticality importance measure of X_i
I_{FV}^i	=	the Fussell–Vesely importance measure of X_i
I_{RAW}^i	=	the risk achievement worth of X_i
IR_{IM}^i	=	the new importance measure of X_i by considering the basic event with uncertain probability
M_i	=	the ith middle event of the fault tree
$R_{\mathrm{IM}}(i)$	=	the importance rank of X_i computed by an importance measure
T	=	the top event of the fault tree
X_i	=	the ith basic event of the fault tree

I. Introduction

AIRCRAFT flap mechanism is an important lifting device, its reliability is vital to the safety of the aircraft at taking off and landing. Statistics show that failures of the aircraft flap mechanism have taken place many times in recent years, for examples, BAE146, Boeing-737-300/500, Boeing-747, Boeing-767 et al. have all encountered failures of the flap mechanism. Although most of these failures caused incidents, some of them did have caused catastrophic accidents. According to the Pareto rule [1], less than 20% of the components failure in the risk model contributes to more than 80% of the total risk. By using proper importance analysis to discriminate the risk-significant events, the risk and the maintenance costs of the system can be reduced, and the system reliability can be improved. Thus, it is an important task to identify and rank the basic events in probabilistic risk assessment of the aircraft flap mechanism.

In general, the importance measures (IMs) used in the probabilistic risk assessment include the Birnbaum importance measure (I_B^i) [2], the criticality importance factor (I_{cr}^i) [3], the Fussell–Vesely (I_{FV}^i) [4,5], and the risk achievement worth (I_{RAW}^i) [5]. Against the drawbacks and limitations of these IMs [3], Borgonovo et al. [6,7] defined the differential IM (DIM_i) to evaluate the contribution of the basic event to the top event.

All traditional IMs, including I_B^i, I_{cr}^i, I_{FV}^i, I_{RAW}^i and DIM_i, focus on the importance analysis on the assumption of the basic event with certain probability. In fact, uncertainty exists universally. In the probabilistic risk assessment of the aircraft flap mechanism, the probabilities of the basic events are usually uncertain due to some uncontrollable factors coming from system, circumstance and human factors, and so on. For the sake of expression in the text, the uncertain probability is denoted by *UC-probability*, and the certain probability is denoted by *C-probability*. The probability uncertainty of the basic events is obviously propagated to the top event through the system model, which results in the uncertainties of the IMs and the importance orders of the basic events. In the case of the basic events with the uncertain probabilities, Aven and Nøkland [8] incorporated the traditional IM based on the C-probability and uncertain IM on the variance-based IM to evaluate the effects of the basic events on the probability and on the global uncertainty of the top event, respectively. However, the incorporation lacks the direct contribution of the UC-probability of the basic events to the top event. Youngblood [9] explained the effect and meaning of the robustness of the IM in case of the basic events with UC-probability. Baraldi et al. [10] and Modarres [11] researched the direct effects of the basic events with UC-probability on the top event, and they identified the importance orders of the basic events with UC-probability by a relative ordering method, their method requires a prior assumed importance orders of the basic events, on which the

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importance of the basic events is determined by the relative comparison one to one. The importance orders of the basic events may be affected by the prior assumed importance orders.

In this contribution, a new IM by considering order distribution related to the UC-probability of the basic event is proposed to analyze the unilateral asymmetric movement failure of the aircraft flap mechanism. The new IM can directly identify the significant basic events affecting the top event most. Not only the definition of the new IM is explained, but also the solution of the new IM is discussed. Since the Monte Carlo simulation based universal solution requires too much computational efforts to the complex system and tests, the probability density evolution method (PDEM) used in stochastic dynamic response analysis [12,13] is introduced to solve the new IM, which can improve the computational efficiency greatly while keeping acceptable precision.

II. Description of the Unilateral Asymmetric Movement Failure of the Aircraft Flap Mechanism

The unilateral asymmetric movement failure of the aircraft flap mechanism might result in serious accident, it is difficult to analyze its failure due to a large number of subsystems and components involved. There are two possibilities about the unilateral asymmetric movement failure of the aircraft flap mechanism, the left flap mechanism works while the right one fails, and the reverse. Generally, the left flap and the right one are symmetrical. Assume the asymmetric movement failure probability of one side flap is P, then the reliability of the other flap is $1 - P$, and the unilateral asymmetric movement failure probability of the flap mechanism is $2P(1 - P)$. To simplify the fault tree and reduce the magnitude of the fault tree, the unilateral asymmetric movement failure of the aircraft flap is selected as the top event of the flap mechanism. The connections of the flap mechanism system are listed as follows. The inner flap of aircraft is driven by the no. 1 and no. 2 flap actuators, and no tilt sensor is installed for two actuators to monitor the tilting angle of the inner flap. The outer flap is driven by the no. 3 and no. 4 flap actuators, and the tilt sensors are installed for the no. 3 and no. 4 actuators to monitor the titling angle of the outer flap. Two position sensors to monitor the unilateral flap position are installed near the torque tube components placed out of the flap transmission mechanism. The flap position control device is redundant, which is composed by the no. 1 and no. 2 control units. And the position control system can insulate the signs of the fault control units. If monitoring signs show that the flap is tilt or asymmetry, the control device will stop the motion of the flap and keep the flap in a safety domain immediately.

Figure 1 shows the fault tree of the unilateral asymmetric movement failure of the aircraft flap mechanism.

T is the top event in Fig. 1, namely the unilateral asymmetric movement failure of the aircraft flap mechanism, and $M_i (i = 1, 2, \ldots, 8)$ are the middle events, and $X_i (i = 1, 2, \ldots, 12)$are middle basic events. The detailed meanings of M_i and X_i are listed in Table 1.

III. New IM of the Basic Event with UC-probability and its Solutions

A. IMs of the Basic Event with C-probability

Several IMs are extensively used in the probabilistic risk assessment, such as the Birnbaum importance measure (I_B^i) [2], the criticality importance factor (I_{cr}^i) [3], the Fussell–Vesely (I_{FV}^i) [4,5], and the risk achievement worth (I_{RAW}^i) [5]. These IMs reflect the contribution of the basic event with C-probability to the probability of the top event, and these IMs have their application fields. Under some conditions these IMs also can be transformed to each other. However, there are some shortcomings in these IMs, for example, all these IMs evaluate the risk changes only at the extrema (0,1) of the defined range of the basic event probability, and they do not possess additivity, either. Against the shortcomings of these IMs, Borgonovo et al. [6,7] proposed the differential IM (DIM) to evaluate the contribution of the basic events to the top event.

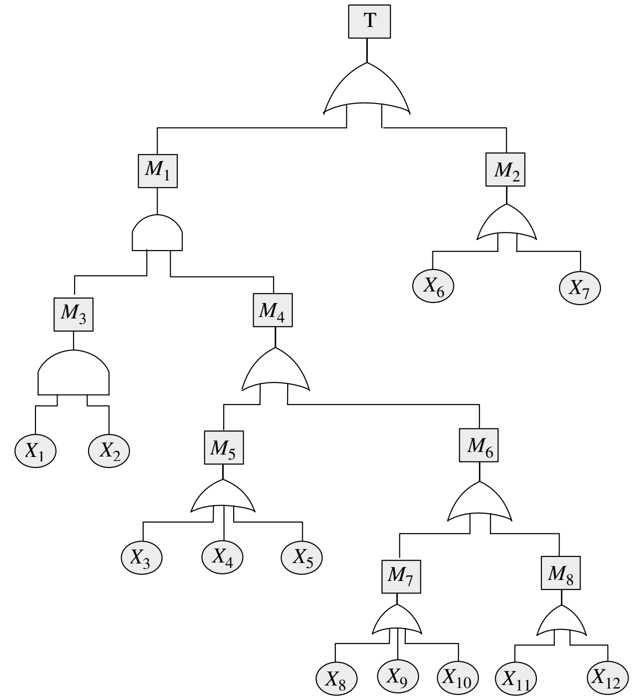


Fig. 1 The fault tree of the unilateral asymmetric movement failure of the aircraft flap mechanism.

The DIM is defined as that in which the probability of the top event changes with respect to the probability of the individual basic event, namely [6]

$$\mathrm{DIM}_i = \frac{\partial Y / \partial X_i}{\sum_{s=1}^{n} \partial Y / \partial X_s} \tag{1}$$

Table 1 Events in the fault tree of the aircraft flap mechanism

Code	Explanation of the event code
M_1	The malfunction of the unilateral transmission and the fault of monitoring function.
M_2	The fault of the branch transmission route of inner flap.
M_3	The malfunction of function monitoring the asymmetric movement.
M_4	The fault of the unilateral transmission.
M_5	The fault of the global transmission route.
M_6	The fault of the branch transmission route of outer flap.
M_7	The fault of the branch transmission route along wingspan direction.
M_8	The fault of the branch transmission route along wing chord direction.
X_1	The fault of monitoring function of no. 1 flap control unit (include the sensor).
X_2	The fault of monitoring function of no. 2 flap control unit (include the sensor).
X_3	The fault of the torque tube between the flap driven device and the no. 1 flap actuator.
X_4	The transmission fault of two 105° gear boxes.
X_5	The mechanical fault of the no. 1 flap actuator.
X_6	The transmission fault of the no. 1 flap actuator in wing chord direction.
X_7	The transmission fault of the no. 2 flap actuator in wing chord direction.
X_8	The transmission fault of the no. 2 flap actuator in wingspan direction.
X_9	The transmission fault of the torque tube between the no. 2 and the no. 3 flap actuators.
X_{10}	The transmission fault of the 161° degree gear box.
X_{11}	The transmission fault of no. 3 flap actuator in wing chord direction.
X_{12}	The transmission fault of no. 4 flap actuator in wing chord direction.

where $\partial Y/\partial X_i$ is the derivative of the top event probability Y with respect to the basic event probability X_i.

Borgonovo et al. [6,7] showed the properties of the DIM in the following:

Property 1: additivity:

$$\mathrm{DIM}_{j,k,\ldots,l} = \mathrm{DIM}_j + \mathrm{DIM}_k + \ldots + \mathrm{DIM}_l \tag{2}$$

Property 2: the sum of the DIMs of all basic events equal unity:

$$\mathrm{DIM}_1 + \mathrm{DIM}_2 + \ldots + \mathrm{DIM}_n = 1 \tag{3}$$

In practice, DIM can be defined as $\mathrm{DIM}_i^{H_1}$ and $\mathrm{DIM}_i^{H_2}$ under the following cases:

Case 1: consider uniform changes in the parameters (H_1):

$$H_1\text{: } \mathrm{d}X_j = \mathrm{d}X_k, \quad \forall\ j,k = 1,2,\ldots,n$$

Case 2: consider proportional relative changes in the parameters (H_2):

$$H_2\text{: } \frac{\mathrm{d}X_j}{X_j} = \frac{\mathrm{d}X_k}{X_k}, \quad \forall\ j,k = 1,2,\ldots,n$$

Some conclusions can be obtained easily, such as

$$\mathrm{DIM}_i^{H_1} \propto I_B^i \qquad \mathrm{DIM}_i^{H_2} \propto I_{\mathrm{cr}}^i, \qquad \mathrm{DIM}_i^{H_2} \propto I_{\mathrm{FV}}^i$$

which means that $\mathrm{DIM}_i^{H_1}$ and I_B^i both reflect the impacts of basic events on the probability of the top event under the assumption that all probabilities of basic events are varied by the same (small) quantity, and the orders of input variables based on I_B^i are equivalent to those on $\mathrm{DIM}_i^{H_1}$. On the other side, that $\mathrm{DIM}_i^{H_2}$, I_{cr}^i and I_{FV}^i reflect that the impacts of the basic events on the probability of the top event under the assumption that all basic event probabilities are varied by the proportional changes, so the orders of the input events based on these three IMs are consistent. However, one should note that $\mathrm{DIM}_i^{H_1}$, $\mathrm{DIM}_i^{H_2}$ and the risk achievement worth I_{RAW}^i may produce different basic event orders because they have no obvious linear relations.

B. Definition of the New IM of the Basic Event with UC-probability

When the probabilities of the basic events are uncertain, the traditional IMs including I_B^i, I_{cr}^i, I_{FV}^i, I_{RAW}^i and DIM_i cannot sufficiently consider the effect of the uncertain probability distribution of the basic event on the top event. Therefore, based on the probabilistic distribution of the UC-probability of the basic event, a new IM is proposed to evaluate the contribution of the basic events with UC-probabilities on the probability of the top event. The new IM can be named as the ordering IM and it is defined by IR_{IM}^i in Eq. (4):

$$IR_{\mathrm{IM}}^i = \frac{E[R_{\mathrm{IM}}(i)]}{\sum_{k=1}^n E[R_{\mathrm{IM}}(k)]} \tag{4}$$

where $R_{\mathrm{IM}}(i)$ is the importance order of the basic event X_i computed by one of the traditional IM.

To satisfy the general rule that the more important of the basic event is, the bigger $R_{\mathrm{IM}}(i)$ is, the ordering IM IR_{IM}^i is sorted ascendingly. To compare each other conveniently, the ordering IM should be normalized in the interval [0, 1]. The normalized quantity, i.e., the denominator of Eq. (4), is the sum of the expectation of the importance orders of all basic events.

The expectation is an important characteristic of the distribution variable, it is simple and popular for engineering technician. Since the uncertainty of the basic event probability is considered, $R_{\mathrm{IM}}(i)$ related to the UC-probability is uncertain and it should be described by distribution function, and $E[R_{\mathrm{IM}}(i)]$ is the expectation of the distribution function of $R_{\mathrm{IM}}(i)$. The expected value of the importance order distribution resulted from the UC-probability of the basic event is used to reflect the effect on the probability of the top event. If the 95% percentile is considered as the new IM characteristic, the ranking determined by the 95% percentile might be different from that determined by the expectation. Two rankings reflect the different perspectives of the distribution of $R_{\mathrm{IM}}(i)$, the integrated information of the uncertainty of $R_{\mathrm{IM}}(i)$ should be expressed by the distribution function, which is inconvenient to comparison.

C. Solution of the new IM Based on the Monte Carlo Method

The Monte Carlo simulation (MCS) is widely applied in importance analysis because of its simple programming and strong ability to high precision. The computational flow for the new IM analysis by the MCS is provided as follows:

1) Sample the kth sample $\{X_{k1}, X_{k2}, \ldots, X_{kn}\}$ for the n-dimensional basic event $\{X_1, X_2, \ldots, X_n\}$ according to the distribution of the uncertain probability of the basic event.

2) Substitute the kth sample into the probabilistic risk assessment model, and compute the kth IMs of the basic events corresponding to $\{X_{k1}, X_{k2}, \ldots, X_{kn}\}$ by use of a traditional IM definition.

3) Sort ascendingly the IMs and obtain the kth importance orders $R_{\mathrm{IM}}^k(i)$ of each basic event.

4) Repeat the preceding processes N times, and fit the probability density functions (PDFs) of $R_{\mathrm{IM}}(i)$.

5) Solve the new IMs by use of Eq. (4).

The preceding MCS for the new IM is very simple and easy to be implemented, but its computational effort for a complicate model usually cannot satisfy the engineering requirement, therefore a new method based on PDEM is provided in the following subsection.

D. Solution of the new IM Based on the PDEM

From the preceding content, we can find that solving the PDF of the importance order $R_{\mathrm{IM}}(i)$ is crucial to obtain the new IM. Recently developed PDEM can provide a highly efficient method to solve the PDF of $R_{\mathrm{IM}}(i)$. The PDEM can build the essential relation between the stochastic origin and the random phenomenon properly, and obtain the PDF of the stochastic phenomenon by a series of differential computation [14]. Concretely, the following content lists the basic idea of applying the PDEM into solving the new IM.

The PDEM is applied in solving the PDF of dynamic stochastic response firstly. Because the uncertainty of the importance order of the basic event is independent of time, a virtual stochastic process with τ as the virtual time can be constructed as following:

$$Z(\tau) = \varphi(R_{\mathrm{IM}}(i), \tau) = \phi(\boldsymbol{X}, \tau) \tag{5}$$

Since $R_{\mathrm{IM}}(i)$ can be obtained by computing its traditional IM, it is a function of the probabilities $\boldsymbol{X}$ of the basic events. The virtual stochastic process satisfies conditions $Z(\tau)|_{\tau=0} = 0$ and $Z(\tau)_{|\tau=\tau_c} = R_{\mathrm{IM}}(i)$. Denoting the joint PDF of $(Z(\tau), \boldsymbol{X})$ as $f_{Z\boldsymbol{X}}(z, \boldsymbol{x}, \tau)$, and the generalized probability density evolution equation can be constructed by Eq. (6) according to the probability conservation law [15]:

$$\frac{\partial f_{ZX}(z, \boldsymbol{x}, \tau)}{\partial \tau} + \dot{\phi}(\boldsymbol{x}, \tau) \frac{\partial f_{ZX}(z, \boldsymbol{x}, \tau)}{\partial z} = 0 \tag{6}$$

where $\dot{\phi}(\boldsymbol{x}, \tau) = \frac{\partial \phi(\boldsymbol{x}, \tau)}{\partial \tau}$, and the initial condition is

$$f_{ZX}(z, \boldsymbol{x}, \tau)|_{\tau=0} = \delta(z) f_X(\boldsymbol{x}) \tag{7}$$

where $\delta(\cdot)$ is the Dirac function, $f_X(\boldsymbol{x})$ is the joint PDF of the UC-probabilities $\boldsymbol{X}$ of the basic events.

After $f_{ZX}(y, \boldsymbol{x}, \tau)$ is obtained by solving the preceding partial derivative equation, the PDF $f_Z(z, \tau)$ of Z can be obtained by Eq. (8):

$$f_Z(z, \tau) = \int_{\Omega_X} f_{ZX}(z, \boldsymbol{x}, \tau)\, \mathrm{d}\boldsymbol{x} \tag{8}$$

where Ω_X is the variables space in which the UC-probabilities $\boldsymbol{X}$ of the basic events take their values.

Since $Z(\tau_c) = R_{\mathrm{IM}}(i)$, the PDF $f_{R_i}(R_i)$ of $R_{\mathrm{IM}}(i)$ can be computed by Eq. (9):

Table 2 The traditional IMs of the basic events with given C-probabilities of failure

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Failure probability[a]	4.6E − 3	4.6E − 3	4E − 4	5.8E − 2	4E − 2	18E − 6	1.8E − 6	3.5E − 4	4.5E − 2	8.5E − 2	2.5E − 6	2.5E − 6
I_B^i	1.01E − 3	1.01E − 3	2.12E − 5	2.12E − 5	2.12E − 5	1	1	2.12E − 5	2.12E − 5	2.12E − 5	2.12E − 5	2.12E − 5
I_{cr}^i	0.573	0.573	0.001	0.145	0.100	0.213	0.213	0.009	0.113	0.213	6.27E − 6	6.27E − 6*l*
I_{FV}^i	0.573	0.573	0.001	0.145	0.100	0.213	0.213	0.009	0.113	0.213	6.27E − 6	6.27E − 6*l*
I_{RAW}^i	125.1	125.1	3.51	3.36	3.41	1.18E5	1.18E5	3.51	3.39	3.29	3.51	3.51
$DIM_i^{H_1}$	5.25E − 4	5.25E − 4	1.06E − 5	1.06E − 5	1.06E − 5	0.499	0. 499	1.06E − 5	1.06E − 5	1.06E − 5	1.06E − 5	1.06E − 5
$DIM_i^{H_2}$	0.2671	0.2671	4.6E − 4	0.0677	0.0467	0.0993	0.0993	4.09E − 4	0.0525	0.0993	2.92E − 6	2.92E − 6

[a]The failure probabilities are only references of the basic events in the table are not true value.

$$f_{R_i}(R_i) = f_Z(z,\tau)|_{\tau=\tau_c} \tag{9}$$

Then the expectation of $R_{\mathrm{IM}}(i)$ can be obtained by Eq. (10):

$$E(R_{\mathrm{IM}}(i)) = \int_{-\infty}^{+\infty} R_i f_{R_i}(R_i)\,\mathrm{d}R_i \tag{10}$$

By the definition shown in Eq. (4) and using Eq. (10), the new IMs can be computed.

In computational fluid dynamics [16], there are many algorithms to solve hyperbolic conservational partial derivative equations like Eq. (6). Among the strategies of selecting representative points of $\boldsymbol{x}$, the number-theoretical method can improve computational efficiency by reducing the dimensions of the input variables [17,18]. In the finite difference method for solving the equation, the difference scheme with TVD nature or combined difference scheme can obtain more precise result and reduce oscillatory and dissipation in the process of difference [15].

IV. Importance Analysis of the Unilateral Asymmetric Movement Failure of the Aircraft Flap

A. Importance Analysis of the Basic Events with C-probability

Given the probabilities of the basic events of the unilateral asymmetric movement failure of the aircraft flap mechanism as certain values, the results of the importance analysis using the traditional IMs are listed in Table 2.

Some conclusions can be drawn from the preceding results. Firstly, according to the results of I_B^i, I_{RAW}^i and $\mathrm{DIM}_i^{H_1}$, the importance orders of the basic events are $\{X_6, X_7\} > \{X_1, X_2\} > \{X_3, X_4, X_5, X_8, X_9, X_{10}, X_{11}, X_{12}\}$. Secondly, according to the values of I_{cr}^i, I_{FV}^i and $\mathrm{DIM}_i^{H_2}$, the basic events can be classified into four groups, namely $\{X_1, X_2\}$ are the most important basic events, $\{X_6, X_7, X_{10}\}$ and $\{X_4, X_9, X_5\}$ are next, and the left events are least important. Through the importance analysis of the basic event with C-probability, one can find that the transmission faults of the no. 1 and no. 2 flap actuators in wing chord direction and the monitoring function faults of the no. 1 and no. 2 flap control units should be paid more attention to, and so is the transmission fault of the 161° angle gear box.

Since two DIMs have better properties than the other traditional IMs, the important orders and the ordering IMs of the basic events with UC-probabilities are researched on DIM in the following section. The ordering IMs are classified into two types, i.e. $IR^i_{\mathrm{DIM}^{H1}}$ and $IR^i_{\mathrm{DIM}^{H2}}$ corresponding to $\mathrm{DIM}_i^{H_1}$ and $\mathrm{DIM}_i^{H_2}$ respectively.

B. Importance Analysis of the Basic Events with UC-probabilities

In this subsection, the uncertainty of the basic events probability of the aircraft flap mechanism is taken into account. Their probabilities are assumed to follow logarithm normal distribution. The probabilities of the basic events shown in Table 2 are assumed as their nominal values, and their error factors are assumed as 2.0, then the importance analysis of the basic events with UC-probabilities can be solved.

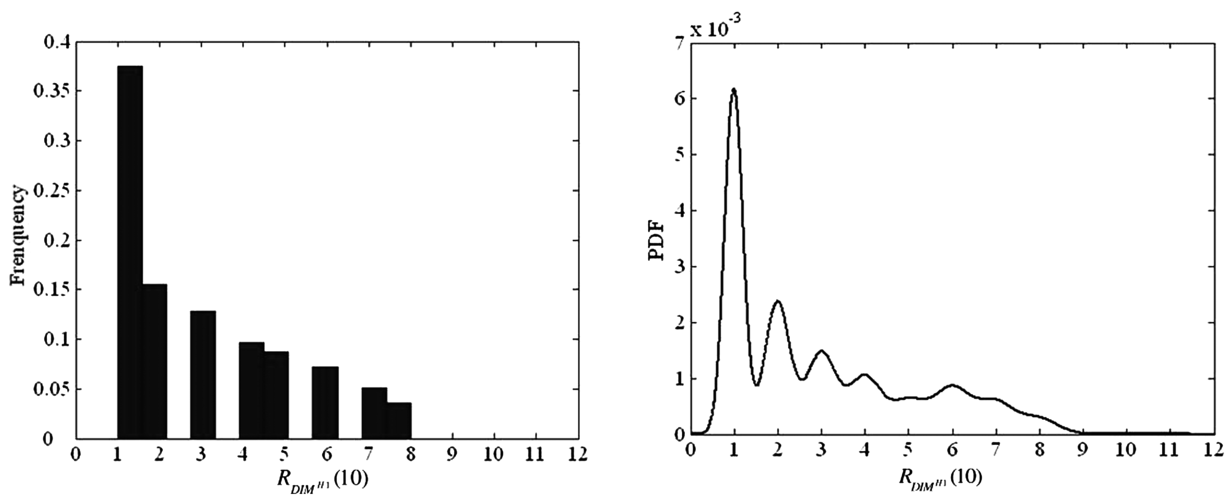
Figures 2 and 3 show the ordering distributions of the basic event X_{10} according to $\mathrm{DIM}_{10}^{H_1}$ and $\mathrm{DIM}_{10}^{H_2}$ computed by the MCS and the PDEM.

The computational results show that two methods can get consistent results, but their computational costs differ largely. For example, obtaining the ordering distribution of one event only needs 442 samples by the PDEM, that needs above 10,000 samples by the MCS. If the uncertainty of the basic events probabilities is considered and the error factors are assumed as 2.0, it can be found by comparing results in Tables 2 and 3 that the ordering IMs of the basic events according to $\mathrm{DIM}_i^{H_1}$ are consistent with the results of the traditional IM by considering the basic events with C-probabilities, while the ordering IMs of the basic events according to $\mathrm{DIM}_i^{H_2}$ are different largely from the results of the traditional IM. The new IM based on $\mathrm{DIM}_i^{H_2}$ shows that $\{X_1, X_2\}$ are the most important basic events, and next are $\{X_6, X_7\}$, others are the least important basic events.

Table 3 Results of the ordering IMs as 2.0 error factors

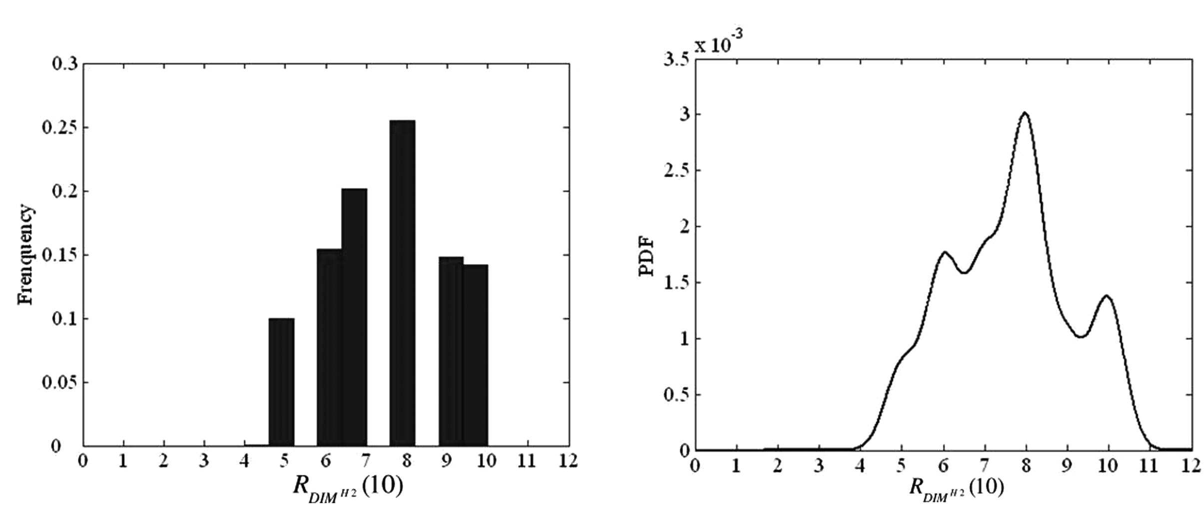
Basic event	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Error factor	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
					$IR^i_{\mathrm{DIM}^{H1}}$							
PDEM	0.143	0.143	0.046	0.042	0.047	0.175	0.174	0.046	0.043	0.045	0.047	0.048
MCS	0.143	0.143	0.046	0.045	0.045	0.178	0.179	0.044	0.044	0.045	0.045	0.046
					$IR^i_{\mathrm{DIM}^{H2}}$							
PDEM	0.134	0.134	0.047	0.093	0.087	0.117	0.117	0.044	0.088	0.099	0.019	0.019
MCS	0.136	0.136	0.0455	0.091	0.0837	0.121	0.120	0.044	0.086	0.098	0.019	0.019

Comparing with the results of the traditional IM, the importance of the basic event X_{10} declines because of the uncertainty probabilities of the basic events, which cannot be estimated by the traditional IMs. Furthermore, if the uncertainty of the basic event probability increases as shown in Table 4, the results of the new IM would be more different from the results obtained by the traditional IMs. With the uncertainty of the basic event probability increasing, the ordering result according to $\mathrm{DIM}_i^{H_1}$ is $\{X_6, X_7\} > \{X_2\} > \{X_1\} > \{X_3, X_4, X_{12}, X_5, X_7, X_8, X_9, X_{10}, X_{11}\}$, and the result according to $\mathrm{DIM}_i^{H_2}$ is $\{X_1, X_2, X_7\} > \{X_6\} > \{X_9, X_{10}\} > \{X_4, X_5\} > \{X_3, X_8, X_{11}, X_{12}\}$, which are shown in Table 4. Therefore, the results of importance analysis for the basic events with C-probabilities should be modified in case that the uncertainty of the basic event probability is considered. The uncertainty of the basic event probability has significant effects on the importance orders of the basic events in probabilistic risk assessment, and the new IM can identify and order the basic events rapidly and properly by taking the UC-probabilities of the basic events into account.



a) MCS b) PDEM

Fig. 2 The ordering distributions of DIM^{H_1} of the basic event X_{10} computed by two methods.



a) MCS b) PDEM

Fig. 3 The ordering distributions of DIM^{H_2} of the basic event X_{10} computed by two methods.

Table 4 Results of the ordering importance measures with the error factor changed

Basic event	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Error factor	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0	2.0	5.0
						$IR^i_{\mathrm{DIM}^{H2}}$						
PDEM	0.142	0.147	0.045	0.044	0.046	0.179	0.178	0.044	0.042	0.043	0.047	0.042
MCS	0.143	0.149	0.044	0.043	0.043	0.182	0.182	0.042	0.042	0.042	0.042	0.043
						$IR^i_{\mathrm{DIM}^{H1}}$						
PDEM	0.132	0.132	0.048	0.087	0.089	0.118	0.131	0.042	0.091	0.091	0.021	0.018
MCS	0.132	0.132	0.048	0.085	0.088	0.119	0.132	0.043	0.091	0.091	0.022	0.018

V. Conclusions

In probabilistic risk assessment, due to the influence of system, environment and the human factors, most of the probabilities of the basic events are uncertain. Traditional IMs are not proper and sometimes disagree from the facts because they do not consider the uncertainty of the basic event probability.

On the traditional importance analyses, the new IM for the basic event with UC-probability is proposed to identify the importance of the basic event in the unilateral asymmetric movement failure of the aircraft flap. The importance analyses are implemented under conditions with certain and uncertain probabilities of the basic events. The results show that the uncertainty of the basic event probability can influence the importance orders of the basic events, while the traditional importance analyses can not effectively reflect the effects of the uncertainty of the basic event probability on the top event. The proposed new IM provides the theoretical basement to identify and order the basic events with UC-probabilities in engineering.

About the solution methods of the new IM, the MCS can be used conveniently, but its computational effort is so heavy that it is not suitable to analyze complex and high expensive model. The presented PDEM can obtain the PDF of the importance orders of the basic events with much less computation costs, thus it provides an effective method to solve the new IM. From the examples one can find that results computed by the PDEM are consistent with those by the MCS, but its computational effort is much saved.

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